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Electron Energy Loss in an InSb Electron-Hole Plasma

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The rate of electron energy loss as a result of inelastic electron-hole scattering in InSb is calculated and compared with the rate of energy loss arising from electron polar phonon scattering. Plasma screening of the interactions, hole drift, and the Kane model of the conduction band are included in the calculation. The results indicate that (i) the electron energy loss rates arising from inelastic electron-hole scattering and electron polar phonon scattering become comparable for a hole density of about 10^{17} cm^{-3} at room temperature, (ii) screening reduces the energy loss rates substantially, and (iii) hole drift in the diffusion approximation has only a very small effect on the energy loss rate.

I. INTRODUCTION

A hole density which is considerably larger than that which occurs in thermal equilibrium may be attained by band-gap impact ionization or injection in many small band-gap semiconductors, such as InSb,[1,2] InAs,[3] and CdTe.[4] Impact ionization and injection should occur in many other III-V and II-VI semiconductors and mixed II-VI compounds. If the current is sufficiently large, the plasma may become magnetically pinched,[1,2] thus further increasing the plasma density.

In these materials, for a small electron and hole density, the most efficient momentum and energy relaxation mechanism for the electrons and holes subject to a high electric field is their interaction with the longitudinal optical (polar) phonons. As the hole density increases, the momentum exchange between electrons and holes becomes an important electronic momentum relaxation mechanism. Furthermore, since the hole mass is finite, the energy exchange between the electrons and holes may provide a mechanism for dissipating the energy supplied to the plasma by an external electric field. An increasing plasma density arising from impact ionization or injection and subsequent current pinching may influence electron scattering in two other ways: First, the electron polar phonon interaction is reduced by screening, which would tend to reduce both electron energy and electron momentum loss to the lattice; second, the hole drift momentum in the external electric field may influence the rate at which an electron exchanges both energy and momentum with the holes.

The above-mentioned effects must take into account the proper band structure of small band-gap semiconductors, since the conduction band of these materials may be very nonparabolic.

Intuitively, one expects that for sufficiently large hole densities, the effects of electron-hole scattering, hole drift, screening, and band structure to have a substantial influence on the transport properties of the narrow band-gap semiconductors. Furthermore, inelastic electron-hole scattering as a mechanism for electron energy loss may influence the temperature of the electron distribution during the magnetic pinch to the extent of favoring the isothermal[5] over the adiabatic[6] pinch at sufficiently large hole density.

The purpose of this paper is a quantitative investigation of electron-hole scattering as an electron energy and momentum loss mechanism in a small band-gap semiconductor. The effect of hole drift on electron energy and momentum loss is treated along with plasma screening and the proper conduction-band structure. The importance of electron-hole scattering is assessed by a comparison with analogous quantities arising from electron polar phonon scattering. InSb is a representative and widely studied small band-gap semiconductor. The remaining calculations and discussions refer to InSb.

Section II contains both a calculation of the electron energy loss rate resulting from inelastic electron-hole scattering and a comparison with the electron energy loss rate resulting from electron polar phonon scattering. Section III contains a calculation of the electron distribution function for a high electric field wherein both electron-hole and electron polar phonon scattering are included. Several simplifying assumptions and approximations are made in order to make the calculations tractable. They are (a) Debye screening of the electron-hole and electron polar phonon interaction. This assumption is necessary to facilitate the calculation, and because an exact treatment of screening in a nonequilibrium plasma is not known. (b) The holes occupy a spherical parabolic band. According to Kane[7] the heavy hole band is parabolic, but the constant-energy surfaces are not spherical. The nonsphericity of the heavy hole band is assumed to have a negligible effect on the properties of the conduction electrons arising from electron-hole scattering. (c) The electron-hole interaction does not affect the hole distribution function. This assumption is reasonable since the hole polar phonon interaction is much greater than the electron polar phonon interaction in InSb, as evidenced by the much smaller hole mobility. The hole distribution is assumed to be a displaced Maxwellian in the diffusion approximation. (d) The wave functions are calculated by Kane.[7] (e) Both the electron and hole distributions are nondegenerate.

II. ELECTRON ENERGY LOSS RATE

The time rate at which an electron with wave vector $\vec{k}$ and energy $\mathcal{E}$ loses energy to the hole distribution as a result of electron-hole scattering is

$$\left(\frac{\delta\mathcal{E}}{\delta t}\right)_{e\text{-}h}=\sum_{\vec{k}'}\sum_{\vec{k}_h}\sum_{\vec{k}_h'}(\mathcal{E}-\mathcal{E}')P(\vec{k},\vec{k}';\vec{k}_h,\vec{k}_h')g(\vec{k}_h), \tag{1}$$

where $\vec{k}$ and $\vec{k}_h$ describe the initial electron and hole states with energy eigenvalues $\mathcal{E}$ and $\mathcal{E}_h$, and $\vec{k}'$ and $\vec{k}_h'$ describe the electron and hole states of energy $\mathcal{E}'$ and $\mathcal{E}_h'$ in which the electron and hole are found after the scattering event. $g(\vec{k}_h)$ is the hole distribution function, and $P(\vec{k},\vec{k}';\vec{k}_h,\vec{k}_h')$ is the inherent transition rate for a Coulomb interaction in which an electron makes a transition from $\vec{k}$ to $\vec{k}'$ and a hole makes a transition from $\vec{k}_h$ to $\vec{k}_h'$. Both the electron and hole distributions are assumed nondegenerate so that restrictions arising from the Pauli principle need not be imposed on the electronic energy loss rate. The inherent transition rate is given by first-order time-dependent perturbation theory

$$P(\vec{k},\vec{k}';\vec{k}_h,\vec{k}_h')=(2\pi/\hbar)\,|\langle\vec{k}',\vec{k}_h'|\mathcal{H}|\vec{k},\vec{k}_h\rangle|^2 \times\delta(\mathcal{E}+\mathcal{E}_h-\mathcal{E}'-\mathcal{E}_h'), \tag{2}$$

where $\mathcal{H}$ is the screened Coulomb interaction between an electron and a hole. The plasma screening of the interaction is assumed to be described by a Debye screening length which is generalized for application to a two component plasma in which the electrons and the holes are neither in equilibrium with the lattice nor in equilibrium with each other. The possibility for unequal temperatures occurs because the external electric field may heat the electrons and holes differently. The screening length is

$$\lambda=\{(4\pi e^2/k\epsilon_0)\,[(N_e/T_e)+(N_h/T_h)]\}^{-1/2}, \tag{3}$$

where e is the magnitude of the electronic charge. N_e and N_h are the electron and hole densities, respectively; T_e and T_h are the electron and hole temperatures, respectively; ϵ_0 is the static lattice dielectric constant; k is the Boltzmann constant. The electron temperature T_e appearing in the screening length is henceforth treated as a parameter, and no attempt is made to determine T_e self-consistently. The electron-hole interaction is

$$\mathcal{H}=-(e^2/\epsilon_0 r)\,e^{-r/\lambda}, \tag{4}$$

where r is the electron-hole separation.

The wave functions implied in the matrix element of Eq. (2) are those appropriate to the conduction and valence bands of InSb. The valence-band wave functions are taken to be properly normalized plane waves for holes of effective mass m_h^*. The hole energy-momentum dispersion relation is assumed to be spherical and quadratic.

The conduction band is assumed to be nonparabolic but spherical. The electron energy-momentum dispersion relation is written

$$k^2=(2m_n/\hbar^2)\,\gamma(\mathcal{E}), \tag{5}$$

where m_n is the effective mass at the band edge, and γ describes the nonparabolicity. The conduction-band wave functions were calculated by Kane.[7] These eigenfunctions are determined by solving the $\vec{k}\cdot\vec{p}$ and spin-orbit interaction between the valence and conduction bands, but neglecting higher conduction bands. Using these wave functions and $\mathcal{H}$ of Eq. (4), the inherent transition rate becomes

$$P(\vec{k},\vec{k}';\vec{k}_h,\vec{k}_h')=\frac{2\pi}{\hbar}\left(\frac{4\pi e^2}{\epsilon_0}\right) \times\frac{\mathcal{G}(k,k',y)\,\delta(\mathcal{E}+\mathcal{E}_h-\mathcal{E}'-\mathcal{E}_h')}{(\lambda^{-2}+|\vec{k}-\vec{k}'|^2)^2}, \tag{6}$$

where y is the cosine of the angle between $\vec{k}$ and $\vec{k}'$. Here $\mathcal{G}$ describes the admixture of valence-band states into the conduction band. Both γ and $\mathcal{G}$ are functions of electron energy and are defined in terms of the energy-gap and the spin-orbit splitting of the valence bands. The functions were discussed by Matz,[8] and are included in Appendix A. The hole distribution function appearing in Eq. (1) is a drifted Maxwellian in the diffusion approximation

$$g(\vec{k}_h)=N_h\left(\frac{2\pi\hbar^2}{m_h^*kT_h}\right)^{3/2}\left[1+\frac{\hbar^2\vec{k}_h\cdot\vec{k}_0}{m_h^*kT_h}\right] \times\exp\left(-\frac{\hbar^2k_h^2}{2m_h^*kT_h}\right), \tag{7}$$

where $\hbar\vec{k}_0$ is the hole drift momentum.

The details of the calculation of the electron energy loss rate are included in Appendix B. The result is

$$\left(\frac{\delta\mathcal{E}}{\delta t}\right)_{e-h}=\frac{N_h e^4 2\sqrt{2}\pi\epsilon^2}{\epsilon_0^2\sqrt{m_n\gamma}}\left\{\sum_i P_i Q_i G_{i2}\gamma' -\frac{kT_h}{\gamma'}\sum_i \frac{P_i}{Q_i}\left[(\gamma' Q_i)^2 G_{i2}\right]\right\}+\frac{N_h e^4 2\sqrt{2}\pi\epsilon\beta(kT_h)^{1/2}}{\epsilon_0^2\gamma(m_n)^{1/2}}\times\sum_i P_i Q_i G_{i2}\gamma'\cos\theta, \quad (8)$$

where $\beta^2=\hbar^2k_0^2/2m_h^*kT_h$ is the ratio of hole drift energy to hole thermal energy $\gamma'=d\gamma/d\mathcal{E}$, and $\epsilon^2=m_n/m_h^*$. P_i, Q_i, and G_{ij} are discussed in Appendix B.

Higher-order terms in ϵ may be readily obtained. The final term in the energy loss rate depends upon the drift energy and the angle which $\vec{k}_e$ makes with the external electric field. The contribution from this term is positive if $\cos\theta=1$ and, therefore, implies that an electron with velocity in the direction of hole drift has a larger rate of energy loss than an electron with velocity in the direction opposite to that of hole drift. Numerical evaluation of Eq. (8) for InSb indicates that the effect of hole drift on the electronic energy loss rate is negligible compared to the first term in Eq. (8), unless hole streaming occurs (i.e., unless β is not small compared to unity). Hole streaming could occur in InSb since the holes interact very strongly with polar phonons. Equation (8) is not strictly correct for hole streaming, since the hole distribution function given by Eq. (7) is valid in the diffusion approximation. To account for hole streaming, the hole distribution function may be taken to be a displaced Maxwellian without making the diffusion approximation. This approach should be valid for a large hole density. For a small hole density the maximum anisotropy approach of Baraff[9] may be appropriate.

The importance of electron-hole scattering as an electronic energy loss mechanism can be assessed by a comparison of the electronic energy loss rates for electron-hole and electron polar phonon scattering. The interaction of electrons with polar phonons is the dominant electronic energy loss mechanism for electrons with energy greater than the optical phonon energy in InSb, if the hole density is small. For high carrier density, the electron polar phonon interaction is screened by the plasma,[10] and, consequently, the rate of energy loss is reduced. An expression for the energy loss rate in a nonparabolic conduction band has been derived by Conwell.[11] The inclusion of screening and the correct conduction-band wave functions in the calculation of the electronic energy loss rate arising from the interaction with polar phonons yields

$$\left(\frac{\delta\mathcal{E}}{\delta t}\right)_{e-P}=\frac{eF_0k\Theta}{(2m_n\gamma)^{1/2}}\left[\mathfrak{N}\,\Omega_+\,\frac{d\gamma}{d\mathcal{E}}\bigg|_{\mathcal{E}+k\Theta}-(\mathfrak{N}+1)\,\Omega_-\,\frac{d\gamma}{d\mathcal{E}}\bigg|_{\mathcal{E}-k\Theta}H(\mathcal{E}-k\Theta)\right], \quad (9)$$

where $eF_0=(m_ne^2k\Theta/\hbar^2)\,(1/\epsilon_\infty-1/\epsilon_0)$,

$$\mathfrak{N}=[e^{\Theta/T}-1]^{-1},$$

$$\Omega_\pm(\mathcal{E})=\sum_i P_i(\mathcal{E})Q_i(\mathcal{E}\pm k\Theta)\int_{r\pm}^{s\pm}\frac{q^3R_i(q)\,dq}{[q^2+\hbar^2/(2m_n\lambda^2)]^2},$$

$$r_\pm=\left|[\gamma(\mathcal{E})]^{1/2}-[\gamma(\mathcal{E}\pm k\Theta)]^{1/2}\right|,$$

$$s_\pm=[\gamma(\mathcal{E})]^{1/2}+[\gamma(\mathcal{E}\pm k\Theta)]^{1/2}.$$

$\mathfrak{N}$ is the polar phonon occupation number, Θ is the Debye temperature, ϵ_∞ the high-frequency dielectric constant, and

$$\mathcal{H}(\mathcal{E}-k\Theta)=0,\quad \mathcal{E}<k\Theta$$
$$=1,\quad \mathcal{E}\geq k\Theta.$$

P_i, Q_i, and R_i are the same as in the case of electron-hole scattering.

The rates of electron energy loss as determined from Eqs. (8) and (9) are plotted for two different sets of temperatures in Figs. 1 and 2, where the term involving hole drift has been omitted since it is small. For InSb, the following parameters were used[8,12]: $G=0.18$ eV, $\Delta=0.81$ eV, $m_n=0.014$ m, $\Theta=264$ °K, $\epsilon_0=18.7$, $F_0=600$ V/cm, $m_h=0.18$ m. In Fig. 1 the lattice and hole temperatures are 77 °K and the electron temperature is the optical phonon Debye temperature 264 °K. This choice of electron and hole temperatures was made under the assumption that a range of electric fields exists for which the hole transport remains Ohmic but the electrons become hot. In InSb, the electrons avalanche at an electric field of approximately 300 V/cm. At this field the electron distribution is assumed to be heated to an average temperature close to the Debye temperature. An estimate of the heating of the hole distribution at 300 V/cm can be obtained from Stratton.[13] Such an estimate indicates that the hole temperature is not significantly different from its value at zero

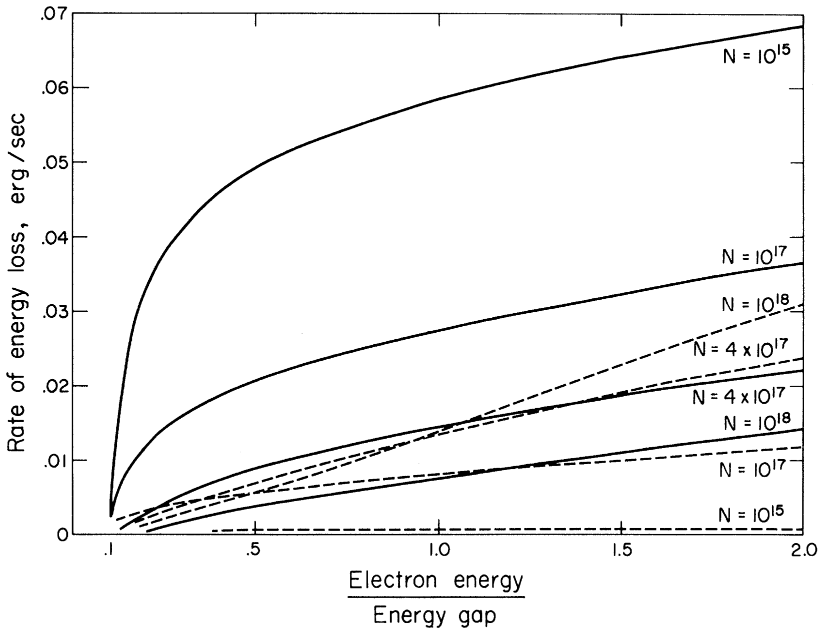

FIG. 1. Rate of electron energy loss to polar phonons (solid curves) and to holes (dashed curves) for various pair densities in InSb. Here $T_e = 264\,°\mathrm{K}$ and $T_L = T_h = 77\,°\mathrm{K}$.

electric field. In Fig. 2 the lattice, hole, and electron temperatures are 300 °K. These choices of the temperature parameters permit a rough comparison of the energy loss rates. In both figures, N is the electron-hole pair density.

Figures 1 and 2 show that plasma screening of the electron polar phonon interaction substantially reduces the rate at which a high-energy electron loses energy to the polar phonons. For an electron with an energy equal to twice the energy gap, the rate of energy loss by polar phonon emission is reduced by a factor of about 3 as the plasma pair density increases from 10^{15} cm^{-3} to 10^{18} cm^{-3}. The rate of electron energy loss by inelastic electron-hole scattering continues to increase as the plasma pair density increases, in spite of the increased plasma screening of the electron-hole interaction. In both Fig. 1 and Fig. 2, the energy loss rates are comparable at a pair density of approximately 4×10^{17} cm^{-3}, although the energy loss rate resulting from inelastic electron-hole scattering increases faster at high electron ener-

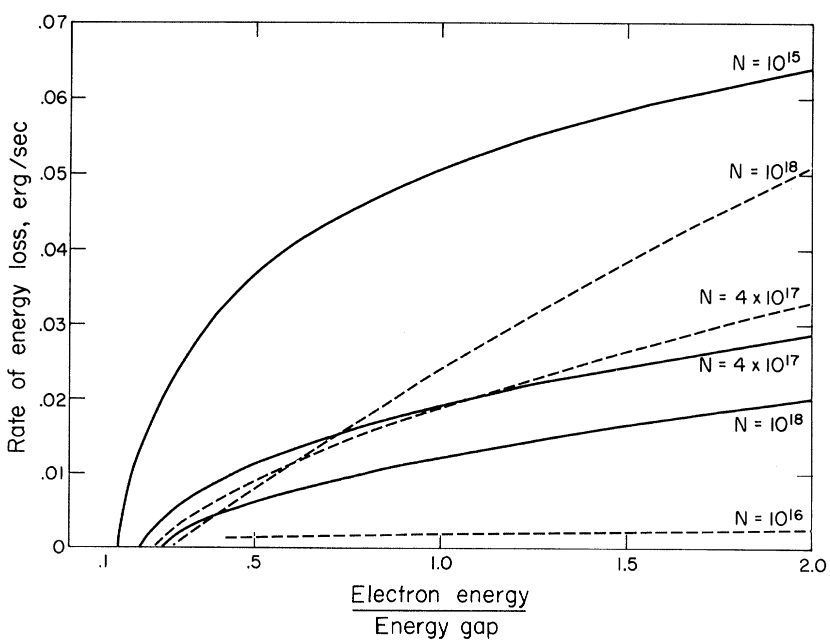

FIG. 2. Rate of electron energy loss to polar phonons (solid curves) and to holes (dashed curves) for various pair densities in InSb. Here $T_e = T_h = T_L = 300\,°\mathrm{K}$.

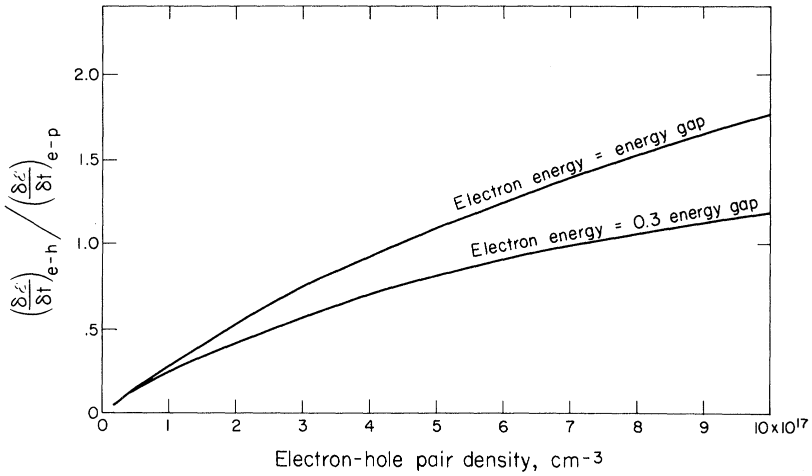

FIG. 3. Ratio of electron energy loss rate by electron-hole scattering to electron energy loss rate by electron polar phonon scattering, versus electron-hole pair density in InSb. Here $T_e = 264\,°K$ and $T_h = T_L = 77\,°K$.

gy. The ratio of the energy loss rate by inelastic electron-hole scattering to the energy loss rate by polar phonon scattering is plotted in Figs. 3 and 4, for the same sets of temperatures as in Figs. 1 and 2, respectively. In both Fig. 3 and Fig. 4, the energy loss rates are equal at a pair density of about 4×10^{17} cm^{-3} for an electron with energy equal to the energy gap. At lower electron energy, the equality of the energy loss rates occurs at a higher pair density.

The energy loss rates plotted in Figs. 1–4 contain the effects of both nonparabolicity of the conduction band and the admixture of valence-band states into the conduction band. The contribution of these two effects to the electron energy loss rate by inelastic electron-hole scattering is depicted in Fig. 5, wherein the energy loss rates are plotted for lattice, hole, and electron temperatures of 300 °K and an electron-hole pair density of 4×10^{17} cm^{-3}. The effect of admixing valence-band states into the conduction band is to decrease the energy loss, and the effect of nonparabolicity is to increase the rate at which a high-energy electron loses energy via inelastic electron-hole scattering.

III. DISTRIBUTION FUNCTION

The preceding comparison of the energy loss rates demonstrates that, at sufficiently high hole density in InSb, inelastic electron-hole scattering becomes comparable to polar phonon emission as an electronic energy loss mechanism. An alternative method of comparing the effectiveness of electron-hole and electron polar phonon scattering in cooling the electron distribution is to calculate the high electric field electron distribution function, accounting for electron-hole and electron polar phonon scattering. The symmetric part of the distribution function describes the electronic energy distribution and, therefore, indicates the effectiveness of the scattering mechanisms in dissipating the energy supplied by the external electric field.

The effect of electron-hole scattering on the

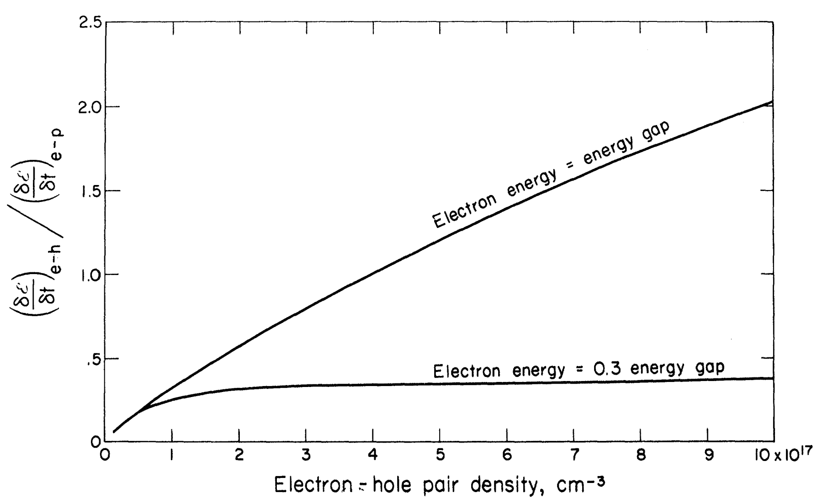

FIG. 4. Ratio of electron energy loss rate by electron-hole scattering to electron energy loss rate by electron polar phonon scattering, versus electron-hole pair density in InSb. Here $T_e = T_h = T_L = 300\,°K$.

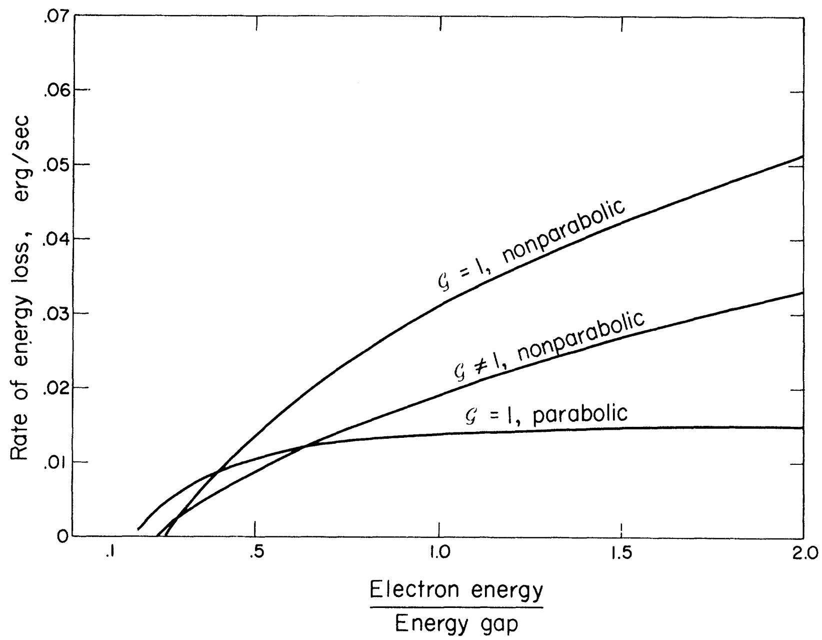

FIG. 5. Effects of nonparabolicity and p-wave admixture on electron energy loss rate arising from electron-hole scattering in InSb. Here $T_e = T_h = T_L = 300\,^\circ\mathrm{K}$, and the pair density is $4\times 10^{17}\ \mathrm{cm}^{-3}$.

high-field electronic distribution function is determined by solving for the distribution function in the unrealistic limit in which all other scattering mechanisms are neglected, and then amending the distribution function to include electron polar phonon scattering. The Boltzmann equation is

$$\left(\frac{\delta f}{\delta t}\right)_F + \left(\frac{\delta f}{\delta t}\right)_{e-h} = 0\ , \tag{10}$$

where the two terms are the rates of change of the distribution function due to the external electric field and electron-hole scattering, respectively. As in the calculation of the energy loss rate, the hole transport properties are presumed to be unaffected by electron-hole scattering, and the conduction-band nonparabolicity and wave functions, screening of the electron-hole interaction by the plasma, and hole drift are included.

The electron-hole collision term is

$$\left(\frac{\delta f}{\delta t}\right)_{e-h} = \sum_{\vec{k}'}\sum_{\vec{k}_h}\sum_{\vec{k}_h'} P(\vec{k}', \vec{k}; \vec{k}_h', \vec{k}_h) f(\vec{k}') g(\vec{k}_h') - \sum_{\vec{k}'}\sum_{\vec{k}_h}\sum_{\vec{k}_h'} P(\vec{k}, \vec{k}'; \vec{k}_h, \vec{k}_h') f(\vec{k}) g(\vec{k}_h)\ , \tag{11}$$

where $P(\vec{k}, \vec{k}'; \vec{k}_h, \vec{k}_h')$ are given by Eq. (6). The Boltzmann equation can be solved in the diffusion approximation wherein

$$f(\vec{k}) = f_0(\mathcal{E}) + f_1(\mathcal{E})\cos\theta\ .$$

The hole distribution function is given by Eq. (7). The details of the calculation of f_0 and f_1 are given in Appendix C. The result is

$$f_0 = \Omega \exp\left[-\int^{\mathcal{E}} \frac{\sum_i P_i Q_i G_{i2}^{e-h}\, d\mathcal{E}}{e^2F^2\sqrt{2}\gamma^{3/2}\tau_{e-h}/[3(m_n)^{1/2}(\gamma')^3] + kT_h \sum_i P_i Q_i G_{i2}^{e-h}}\right]\ , \tag{12}$$

$$f_1 = -\frac{eF\tau_{e-h}}{m_n}\,\frac{(2m_n\gamma)^{1/2}}{\gamma'}\,\frac{df_0}{d\mathcal{E}}\ , \tag{13}$$

$$\tau_{e-h}^{-1} = [\sqrt{2}\,\pi N_h e^4 \gamma' / \epsilon_0^2 (m_n)^{1/2} \gamma^{3/2}] \times \sum_i P_i Q_i G_{i4}\ , \tag{14}$$

$$G_{i2}^{e-h} = [4\pi N_h \epsilon^2 e^4/\epsilon_0^2]\, G_{i2}\ . \tag{15}$$

Ω is the normalization constant. Contributions to f_0 and f_1 which involve hole drift and ϵ^n, $n > 2$ have been omitted, although in principle these can be included. The contributions involving hole drift are small in the hole diffusion approximation and have, therefore, been neglected.

Matz[8] has solved the Boltzmann equation in the diffusion approximation for the spherical part of the distribution function for a high electric field for InSb under the assumption that polar phonons provide the dominant electronic energy and momentum loss mechanism. Modifying his expressions to include screening and then incorporating them into the present calculation yields

$$f_0=\Omega\exp\left[-\int^{\mathcal{E}}\frac{\sum_i P_i Q_i\,(G^{e-h}_{i2}+G^{e-P}_{i2})\,d\mathcal{E}}{[e^2F^2\sqrt{2}\gamma^{3/2}\tau/3(m_n)^{1/2}(\gamma')^3]+\sum_i P_i Q_i(\mu G^{e-P}_{i2}+kT_h G^{e-h}_{i2})}\right], \quad (16)$$

where $\mu=k\Theta(\mathfrak{N}+\frac{1}{2})$, $\tau^{-1}=\tau^{-1}_{e-h}+\tau^{-1}_{e-p}$,

$$G^{e-p}_{i2}=eF_0k\Theta\int_{k\Theta\gamma'/\sqrt{\gamma}}^{2\sqrt{\gamma}}\frac{q^3R_i(q)\,dq}{[q^2+(\hbar^2/2m_n\lambda^2)]^2}, \quad (17)$$

$$\tau^{-1}_{e-p}=\{eF_0(2\mathfrak{N}+1)\gamma'/[2\gamma^{3/2}(2m_n)^{1/2}]\}\sum_i P_i Q_i G_{i4}. \quad (18)$$

The symmetric part of the distribution function f_0 describes the electron energy distribution. The denominator in the integrand of the exponent of e in the expression for f_0 is proportional to the rate at which energy is supplied to the electron distribution by the electric field, phonon absorption, and inelastic electron-hole scattering. The numerator is proportional to the rate at which energy is dissipated by phonon emission and inelastic electron-hole scattering. The relative importance of electron-hole scattering as a cooling mechanism can be assessed by comparing the terms G^{e-h}_{i2} and G^{e-p}_{i2}, which appear in the numerator. At large $\mathcal{E}$,

$$G^{e-p}_{i2}/G^{e-h}_{i2}=eF_0k\Theta\epsilon_0^2/(4\pi N_h\epsilon^2e^4).$$

For InSb, this ratio is equal to unity for a hole density N_h of 3×10^{17} cm^{-3}. Therefore, at this hole density, inelastic electron-hole scattering is as efficient as electron polar phonon scattering in cooling the electron distribution.

To be a valid distribution function, f_0 must be normalizable. For electron-hole and electron polar phonon scattering in a parabolic conduction band, f_0 defined by Eq. (17) cannot be normalized. Therefore, these two scattering mechanisms cannot contain the electron distribution in a parabolic band, and electron runaway occurs. However, the conduction band of InSb is very nonparabolic. If the electron dispersion relation at high electron energy can be represented by $\mathcal{E}\propto k^{\zeta}$, and if electron polar phonon scattering dominates the momentum relaxation time, then f_0 is normalizable for $\zeta<\frac{4}{3}$ and approaches a Maxwellian for $\zeta<1$. If electron-hole scattering dominates the momentum relaxation time, then f_0 is normalizable for $\zeta<\frac{2}{3}$ and approaches a Maxwellian for $\zeta<\frac{1}{2}$. At sufficiently large hole density, the electron energy dissipation could occur by inelastic electron-hole scattering and electron momentum dissipation could occur by electron polar phonon scattering. The degree of nonparabolicity required would be $\zeta=\frac{4}{3}$, which is not unreasonable for InSb.

The preceding discussion of the electron energy loss rate and distribution function indicates that the inelastic nature of the electron-hole collision and plasma screening of the electron polar phonon and electron-hole interactions should be taken into account in calculations of the electron transport properties of InSb if the hole density becomes of order 10^{17} cm^{-3}. The calculation also indicates that hole drift does not affect the electron energy loss or relaxation time if hole streaming does not occur.

Electron-electron scattering is completely ignored in the preceding calculations in order to elucidate the influence of electron-hole scattering on the high electric field electronic distribution function. This assumption is certainly not justified in an electron-hole plasma in which the density of electron-hole pairs is large enough so that electron-hole collisions are an important electronic energy loss mechanism. Since maximum energy exchange in a Coulomb collision is proportional to the ratio of the masses of the particles involved, and if electron-hole collisions are an important energy loss mechanism, then electron-electron collisions must be even more important as an energy loss mechanism for a high-energy electron. Because electron-electron collisions were not included in the calculation, the distribution function is not strictly applicable to a high-density electron-hole plasma such as may occur during impact ionization and subsequent current pinching. The electron distribution function should, however, be applicable to high-field experiments in p-InSb in which the electrons are minority carriers (i.e., where the electron density is sufficiently small that electron-electron scattering is unimportant).

APPENDIX A

The function which describes the admixture of valence-band states into the conduction band is

$$G(k,k',y)=\tfrac{1}{2}\sum_{\mu}\sum_{\mu'}\left|\int d\vec{r}\,\phi^*_{\mu',k'}(\vec{r})\,\phi_{\mu,k}(\vec{r})\right|^2, \quad (A1)$$

where $\phi_{k,\mu}$ are the cell periodic parts of the Bloch functions and μ is a spin eigenvalue. The integration is over a unit cell. The $\phi_{k,\mu}$ are given by Eq. (14) of Ref. 7. The result is

$$G(k, k', y) = a_k^2 a_{k'}^2 + 2a_k a_{k'}(b_k b_{k'} + c_k c_{k'})y + (b_k b_{k'} + c_k c_{k'})^2 y^2 + [\tfrac{1}{4} b_k b_{k'}^2 + \tfrac{1}{2}(b_k c_{k'} + c_k b_{k'})^2 - (1/\sqrt{2})\, b_k b_{k'}(b_k c_{k'} + c_k b_{k'})](1 - y^2) \quad , \tag{A2}$$

where y is the angle between $\vec{k}$ and $\vec{k}'$ and

$$\begin{aligned} a_k^2 &= N^{-2}\mathcal{E}(\mathcal{E}+G)(\mathcal{E}+G+\Delta)(\mathcal{E}+G+\tfrac{2}{3}2\Delta) \,, \\ b_k^2 &= N^{-2}(\tfrac{2}{9})\mathcal{E}^2\Delta^2 \,, \\ c_k^2 &= N^{-2}\mathcal{E}^2(\mathcal{E}+G+\tfrac{2}{3}\Delta) \,, \\ N^2 &= \mathcal{E}(\mathcal{E}+G)(\mathcal{E}+G+\Delta)(\mathcal{E}+G+\tfrac{2}{3}\Delta) + \tfrac{2}{9}\mathcal{E}^2\Delta^2 + \mathcal{E}^2(\mathcal{E}+G+\tfrac{2}{3}\Delta)^2 \,. \end{aligned} \tag{A3}$$

G and Δ are the band-gap and the spin-orbit splitting of the valence band. The function γ which describes the nonparabolicity is

$$\gamma(\mathcal{E}) = \frac{\mathcal{E}(\mathcal{E}+G)(\mathcal{E}+G+\Delta)(G+\tfrac{2}{3}\Delta)}{G(G+\Delta)(\mathcal{E}+G+\tfrac{2}{3}\Delta)} \tag{A4}$$

and is obtained from Eq. (10) of Ref. 7, neglecting terms of order m_n/m, where m is the free-electron mass and m_n is the band-edge effective mass.

APPENDIX B

The rate of energy loss defined by Eq. (1) is calculated as follows: The summation over $\vec{k}_h'$ in Eq. (1) may be immediately performed using conservation of momentum implied by the matrix element, namely, $\vec{k}+\vec{k}_h = \vec{k}'+\vec{k}_h'$ (neglecting umklapp processes). The remaining sums over $\vec{k}'$ and $\vec{k}_h'$ are converted to integrals by $\sum \to (\frac{1}{2\pi})^3 \int$. The result is

$$\left(\frac{\delta\mathcal{E}}{\delta t}\right)_{e-h} = \frac{1}{(2\pi)^6}\,\frac{2\pi}{\hbar}\left(\frac{4\pi e^2}{\epsilon_0}\right)^2 \iint d\vec{k}'\,d\vec{k}_h \times \frac{(\mathcal{E}-\mathcal{E}')\,\mathcal{G}(k,k',y)\,\delta(\mathcal{E}+\mathcal{E}_h-\mathcal{E}'-\mathcal{E}_h')}{(\lambda^{-2}+|\vec{k}-\vec{k}'|^2)^2} \quad . \tag{B1}$$

The integrals over $\vec{k}'$ and $\vec{k}_h$ may be performed after transforming to the sets of variables $(u, q\ \psi)$, and (z, x, ϕ), where

$$u = \mathcal{E}' - \mathcal{E} \quad ,$$

$$q = [\hbar/(2m_n)^{1/2}]\,|\vec{k}-\vec{k}'| \quad , \tag{B2}$$

Ψ is the azimuth of $\vec{k}'$ with respect to $\vec{k}$,

$$z = (\hbar^2 k_h^2/2m_h^*)^{1/2} \quad ,$$

$$x = \cos\{\vec{k}_h, [\hbar/(2m_n)^{1/2}](\vec{k}'-\vec{k})\} \quad ,$$

ϕ is the azimuth of $\vec{k}_h$ with respect to $[\hbar/(2m_n)^{1/2}] \times (\vec{k}'-\vec{k})$. Furthermore, $\mathcal{G}(k, k', y)$ may be expressed in terms of the variables $\mathcal{E}$, u, and q as

$$\mathcal{G}(\mathcal{E}, u, q) = \textstyle\sum_i P_i(\mathcal{E}) Q_i(\mathcal{E}+u) R_i(q) \quad , \tag{B3}$$

where P_i, Q_i, and R_i are obtained by inspection from Eq. (A2). Using Eqs. (7), (B2), and (B3), Eq. (B1) becomes

$$\left(\frac{\delta\mathcal{E}}{\delta t}\right)_{e-h} = \frac{Ne^2(\sqrt{\pi})2^{5/2}}{\epsilon_0^2(m_n\gamma)^{1/2}(kT_h)^{3/2}} \times \iint\iint du\,dq\,dx\,dz\,\delta(u-2\epsilon zqx+\epsilon^2q^2) \times \frac{d\gamma/d\mathcal{E}|_{\mathcal{E}+u}\, qz^2 u e^{-z^2/kT_h} \sum P_i(\mathcal{E})Q_i(\mathcal{E}+u)R_i(q)}{(q^2+\hbar^2/2m_n\lambda^2)^2} \times \left[1+\frac{\beta zx[-q^2-\gamma(\mathcal{E})+\gamma(\mathcal{E}+u)]}{(kT_h)^{1/2}q[\gamma(\mathcal{E})]^{1/2}}\right]\cos\theta \;, \tag{B4}$$

where θ is the angle between $\vec{k}$ and the electric field, $\epsilon^2 = m_n/m_h^*$, and β^2 is the ratio of hole drift energy to hole thermal energy

$$\beta^2 = \hbar^2 k_0^2/2m_h^* kT_h \quad .$$

The integration limits are

$$\begin{gathered} -\mathcal{E} \le u \le \infty \quad , \\ [\gamma(\mathcal{E}+u)]^{1/2} - [\gamma(\mathcal{E})]^{1/2} \le q \le [\gamma(\mathcal{E}+u)]^{1/2} + [\gamma(\mathcal{E})]^{1/2} \,, \\ 0 \le z \le \infty \quad , \\ -1 \le x \le 1 \quad . \end{gathered} \tag{B5}$$

For InSb at 300 °K, $m_n = 0.014$ and $m_h^* = 0.18$ for the heavy hole valence band,[13] so $\epsilon^2 \approx 0.08$. The integrals of Eq. (B4) are performed for the assumption $\epsilon \ll 1$ by expanding the integrand in a Maclaurin series in ϵ. The δ function is the only part of the integrand requiring expansion and is

$$\delta(u-2\epsilon zqx+\epsilon^2q^2) = \delta(u) - 2\epsilon zqx\delta'(u) + \tfrac{1}{2}\epsilon^2[2q^2\delta'(u) + 4z^2q^2x^2\delta''(u)] + \cdots \quad . \tag{B6}$$

The two lowest-order terms contributing to the electronic energy loss rate are proportional to ϵ and ϵ^2 and are

$$\left(\frac{\delta\mathcal{E}}{\delta t}\right)_{e-h} = \frac{N_h e^4 2\sqrt{2}\pi\epsilon^2}{\epsilon_0^2(m_n\gamma)^{1/2}}\left\{\textstyle\sum_i P_iQ_iG_{i2}\gamma' - \frac{kT_h}{\gamma'}\sum_i \frac{P_i}{Q_i}[(\gamma'Q_i)^2G_{i2}]\right\} + \frac{N_h e^4 2\sqrt{2}\pi\epsilon\beta(kT_h)^{1/2}}{\epsilon_0^2\gamma(m_n)^{1/2}}\textstyle\sum_i P_iQ_iG_{i2}\gamma'\cos\theta \;,$$

where $$G_{ij}=\int_0^{2\sqrt{\gamma}} \frac{q^{j+1}R_i(q)dq}{[q^2+\hbar^2/2m_n\lambda^2]^2} . \tag{B8}$$

APPENDIX C

The electron-hole collision integral defined by Eq. (11) is calculated in the diffusion approximation

$$f(\vec{k})=f_0(\mathcal{E})+f_1(\mathcal{E})\cos\theta ,$$

where θ is the angle between $\vec{k}$ and the external electric field. After converting the sums in Eq. (11) to integrals, changing to the variables of Eq. (B2), and using Eqs. (6), (7), and (B3), the electron-hole collision integral becomes

$$\left(\frac{\delta f}{\delta t}\right)_{e-h}=\{N_h e^4 2^{5/2}\sqrt{\pi}/\epsilon_0^2[m_n\gamma(\mathcal{E})]^{1/2}(kT_h)^{5/2}\}\int\int\int\int du\,dz\,dx\,dq$$
$$\times\frac{|d\gamma/d\mathcal{E}|_{\mathcal{E}+u}qz^2e^{-z^2/kT_h}\sum_i P_i(\mathcal{E})Q_i(\mathcal{E}+u)R_i(q)\delta(u-2\epsilon zqx+\epsilon^2q^2)}{[q^2+\hbar^2/2m_n\lambda^2]^2}$$
$$\times\left[[f_0(\mathcal{E}+u)+f_1(\mathcal{E}+u)\cos\theta']\left(1+\frac{2\beta(z^2-2\epsilon zqx+\epsilon^2q^2)^{1/2}}{(kT_h)^{1/2}}\cos\theta_h'\right)\exp\left(\frac{2zxq\epsilon-\epsilon^2q^2}{kT_h}\right)\right.$$
$$\left.-[f_0(\mathcal{E})+f_1(\mathcal{E})\cos\theta]\{1-[2\beta z/(kT_h)^{1/2}]\cos\theta_h\}\right] . \tag{C1}$$

$\theta_h(\theta_h')$ are the angles between $\vec{k}_h(\vec{k}_h')$ and the external electric field, and in terms of the variables of integration are

$$\cos\theta_h=x[\gamma(\mathcal{E}+u)-\gamma(\mathcal{E})-q^2]\cos\theta/[2q(\gamma)^{1/2}\mathcal{E}] \tag{C2}$$

and

$$\cos\theta_h'=\frac{(zx-q\epsilon)[\gamma(\mathcal{E}+u)-\gamma(\mathcal{E})-q^2]\cos\theta}{(z^2-2qzx\epsilon+q^2\epsilon^2)^{1/2}[\gamma(\mathcal{E})]^{1/2}2q} .$$

The integrations over azimuths of $\vec{k}_h$ and $\vec{k}'$ do not contribute additional terms to the collision integral. The integrations in Eq. (C1) are performed after expansion of the integrand in a Maclaurin series in ϵ. The Boltzmann equation is then reduced to the coupled differential equations for f_0 and f_1,

$$-\frac{eF}{3(2m_n\gamma)^{1/2}\gamma'}\frac{d}{d\mathcal{E}}\left(\gamma'f_1\right)=\frac{2\sqrt{2}\pi N_h\epsilon^2e^4}{\epsilon_0^2(m_n\gamma)^{1/2}\gamma'}\sum_i\frac{P_i}{Q_i}\frac{d}{d\mathcal{E}}\left[(\gamma'Q_i)^2G_{i2}\left(f_0+kT_h\frac{df_0}{d\mathcal{E}}\right)\right]$$
$$+\frac{\sqrt{2}\pi N_he^4\epsilon\beta(kT_h)^{1/2}}{12\epsilon_0^2(m_n)^{1/2}}\left\{-\frac{2\gamma'}{\gamma^2f_1}\sum_iP_iQ_iG_{i2}\frac{d}{d\mathcal{E}}(\gamma f_1^2)+\frac{1}{\gamma\gamma'f_1}\sum_i\frac{P_i}{Q_i}\frac{d}{d\mathcal{E}}[\gamma^{-1}(\gamma'Q_i)^2f_1^2G_{i4}]\right\} \tag{C3}$$

and

$$\frac{2eF\gamma}{(2m_n\gamma)^{1/2}}\frac{df_0}{d\mathcal{E}}=\frac{\sqrt{2}\pi N_he^4\gamma'}{\epsilon_0^2(m_n)^{1/2}\gamma^{3/2}}\sum_iP_iQ_iG_{i2}f_1+\frac{\sqrt{2}\pi N_he^4\epsilon(kT_h)^{1/2}\beta\gamma'}{\epsilon_0^2\gamma(m_n)^{1/2}}\sum_iP_iQ_iG_{i2}\frac{df_0}{d\mathcal{E}} . \tag{C4}$$

Contributions to order ϵ^2 are included in Eq. (C3) and to order ϵ in Eq. (C4). The ϵ^2 contribution to Eq. (C4) is numerically smaller than the ϵ^0 term and has, therefore, been omitted. Equations (C3) and (C4) contain the effect of hole drift and in principle they can be solved for f_0 and f_1. (C4) yields an effective relaxation time

$$\tau_{eff}=\chi\,\tau_{e-h} , \tag{C5}$$

where $$\tau_{e-h}^{-1}=\frac{\sqrt{2}\pi N_he^4\gamma'}{\epsilon_0^2(m_n)^{1/2}\gamma^{3/2}}\sum_iP_iQ_iG_{i2} \tag{C6}$$

and $$\chi = 1 - \frac{\pi N_h e^3 (m_n)^{1/2} \mu_h (\gamma')^2 \sum_i P_i Q_i G_{i2}}{\sqrt{2}\, \epsilon_0^2 \gamma^{3/2}} \,. \tag{C7}$$

$\tau_{e\text{-}h}$ is the Brooks-Herring formula modified for correct conduction-band functions and nonparabolicity, and μ_h is the hole mobility. Barrie[14] has derived the Brooks-Herring formula for a nonparabolic energy band, but he has not included the effects of p-wave admixture into the conduction band.

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